Quiz 9 Fri 11/12/2010.

Let $L: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 \\ x_1 + 3x_2 \end{pmatrix}$$

Let $E = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$ be a basis of $\mathbb{R}^2$.

(a) Find Standard matrix representation $A = [L]_{B_0} = [L]_{B_0,B_0}$

where $B_0 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

(b) Let $B = [L]_E = [L]_{E,E}$

Find a non-singular 2×2 matrix S

Such $B = S^{-1} A S$.

(c) Calculate B.

Solution

(a)
$$A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$$

(b)
$$B = [E \to B_0]^{-1} A \underbrace{[E \to B_0]}_{S}$$

$$S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

(c)
$$B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{-2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 5 & 1 \\ -3 & 5 \end{pmatrix}.$$

Quiz 8 ~~Oct~~ Nov 5, 2010

Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 + 3x_3 \\ 2x_1 - x_2 + 4x_3 \\ 3x_1 + x_2 + 7x_3 \end{pmatrix}.$$

(a) Find ~~A~~ A: the standard matrix representation of L, that is $L(x) = Ax \quad \forall x \in \mathbb{R}^3$.

(b) Find the basis and dimension of $R(L)$, the range of L.

Solution

(a) $$A = [L(e_1)\ L(e_2)\ L(e_3)] = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 7 \end{pmatrix}$$

(b) $R(L) =$ column space of A

$$A \xrightarrow[(3) \to (3) - 3(1)]{(2) \to (2) - 2(1)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -2 \\ 0 & -5 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{5} & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ Basis of $R(L) = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\}$

$\dim R(L) = \text{Rank of } A = 2.$

Quiz 7 Fri Oct 29, 2010

Let $E = \{ (1,2)^T, (2,-1)^T \}$ and

$F = \{ (1,1)^T, (0,2)^T \}$ be two bases of $\mathbb{R}^2$.

(a) Find the transition matrix from E to F.

(b) Let $u = 2\begin{pmatrix}1\\2\end{pmatrix} - 3\begin{pmatrix}2\\-1\end{pmatrix}$

Find $[u]_F$: the coordinate vector with respect to F

Solution (a) $B_0 = \{e_1, e_2\}$ standard basis

$$[E \to F] = [B_0 \to F][E \to B_0] = [F \to B_0]^{-1}[E \to B_0]$$

$$= \begin{pmatrix}1 & 0\\1 & 2\end{pmatrix}^{-1}\begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}2 & 0\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\2 & -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}2 & 4\\1 & -3\end{pmatrix}$$

(b) $[u]_E = \begin{pmatrix}2\\-3\end{pmatrix}$

$$[u]_F = [E \to F][u]_E$$

$$= \frac{1}{2}\begin{pmatrix}2 & 4\\1 & -3\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-8\\+11\end{pmatrix} = \begin{pmatrix}-4\\11/2\end{pmatrix}$$

Check: $u = 2\begin{pmatrix}1\\2\end{pmatrix} - 3\begin{pmatrix}2\\-1\end{pmatrix} = -4\begin{pmatrix}1\\1\end{pmatrix} + \frac{11}{2}\begin{pmatrix}0\\2\end{pmatrix}$

$$\begin{pmatrix}-4\\7\end{pmatrix} = \begin{pmatrix}-4\\7\end{pmatrix} \checkmark$$