

**MULTIPLICITIES AND VOLUMES: AN INTERPLAY  
AMONG ALGEBRA, COMBINATORICS, AND  
GEOMETRY**

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**ABSTRACT.** The notion of multiplicity in algebra traces back to the work of Samuel in 1951 in connections with intersection theory of algebraic varieties. Since then, multiple generalizations of Samuel's multiplicity have been introduced and a wide-ranging set of applications have been found to other topics in mathematics such as: mixed volumes in discrete geometry and degrees in algebraic geometry. Multiplicities are closely related to the theory of convex bodies, and this relation is an active research topic lying in the interaction of Commutative Algebra, Combinatorics, and Algebraic Geometry. In this talk we will discuss some of the history of this topic and its applications. We also report on recent results in joint work with Federico Castillo, Yairon Cid-Ruiz, Binglin Li, Fatemeh Mohammadi, and Naizhen Zhang. In one of these results, we show that mixed volumes of arbitrary convex bodies can be interpreted algebraically as mixed multiplicities of graded families of monomial ideals. In another result, we present a complete characterization of the positivity of multidegrees of multi-projective algebraic varieties and establish a combinatorial description using convex geometry. We also use our methods to prove that double Schubert polynomials have Saturated Newton Polytopes, which settles a conjecture by Monical, Tokcan and Yong.