

THE ALGEBRA OF GENERALIZED BARYCENTRIC COORDINATES

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ABSTRACT. Let P_d be a convex polygon with d vertices. The associated Wachspress surface W_d is a fundamental object in approximation theory, defined as the image of the rational map w_d from P^2 to P^{d-1} , determined by the Wachspress barycentric coordinates for P_d . We show w_d is a regular map on a blowup X_d of P^2 , and if $d > 4$ is given by a very ample divisor on X_d , so has a smooth image W_d . We determine generators for the ideal of W_d , and prove that in graded lex order, the initial ideal of $I(W_d)$ is given by a Stanley-Reisner ideal. As a consequence, we show that the associated surface is arithmetically Cohen-Macaulay, of Castelnuovo-Mumford regularity two, and determine all the graded Betti numbers of $I(W_d)$. Joint work with Corey Irving, Santa Clara Univ.