Factorizations of Algebraic Integers, Block Monoids and Additive Number Theory

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Abstract

Let $\mathcal{O}_K$ be the ring of integers in a finite extension of the rationals. Fundamental problems involving the factorizations of elements in $\mathcal{O}_K$ into irreducibles appear early in the Abstract Algebra curriculum. At a basic level, the usual technique for attacking such problems involves using the norm function. There is a much deeper connection between factorizations of elements in $\mathcal{O}_K$ and the class group of $\mathcal{O}_K$. We will explore this connection and show that it easily generalizes to Dedekind and Krull domains. Implicit in this discussion is the introduction of a structure known as a Block Monoid. A well-known theorem of Geroldinger constructs a monoid homomorphism from $\mathcal{O}_K$ to an appropriately chosen Block Monoid $\mathcal{B}$ which preserves lengths of factorizations of elements into products of irreducibles. The analysis of the factorization properties of Block Monoids leads to the study of two well-known arithmetic constants from Additive Number Theory, the Davenport Constant and the Cross Number.

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