Algebraic Methods for Parameterized Linear Codes

Rafael Villarreal
Instituto Politecnico Nacional (CINVESTAV)

ABSTRACT

Let K be a finite field with q elements and let X be a subset of a projective space $\mathbb{P}^{s-1}$, over the field K, which is parameterized by monomials. We introduce the class of parameterized linear codes arising from X and present algebraic methods to compute their dimensions and lengths. Using commutative algebra and lattice theory, we study the structure of the graded ideal $I(X) \subset S := K[t_1, \ldots, t_s]$ generated by the homogeneous polynomials of S that vanish on X. It is shown that $I(X)$ is a lattice ideal. We give means to compute and study the Hilbert function, the degree, and the regularity of $S/I(X)$. For a parameterized code arising from a connected graph or clutter we are able to compute its length and to determine when $I(X)$ is a complete intersection. A sufficient condition is given for X to be a projective variety defined by binomials and a finite Nullstellensatz is brought up in this connection.