EDGE IDEALS, COVER IDEALS, AND ASSOCIATED PRIMES

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ABSTRACT. Given a simple graph G on vertices $\{x_1, \ldots, x_n\}$, one can form the edge ideal $I(G) \subset k[x_1, \ldots, x_n]$, which is generated by all monomials $x_i x_j$ such that $\{x_i, x_j\}$ is an edge of G. The cover ideal of G, which is the Alexander dual of $I(G)$, is generated by monomials that correspond to vertex covers of G, sets of vertices that contain at least one vertex from each edge. These two ideals have been the focus of much work in combinatorial commutative algebra. We will give an overview of some of the different flavors of results that researchers in both algebra and combinatorics have proven, mentioning some open problems along the way. We will conclude by discussing some recent joint work with Huy Tài Hà and Adam Van Tuyl on associated primes of powers of cover ideals and the graph-theoretic information these associated primes provide.