EXAM

Exam 3, Takehome Exam

Math 3350, Summer II, 2013

August 2, 2013

- This is a Take-home exam.
- Write all of your answers on separate sheets of paper. You can keep the exam questions. You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise instructed, you can use a calculator to do integrals and partial fractions decompositions. State clearly what you are going to put into the calculator and what you got out.
- You can use the textbook and your notes. You can discuss the problems with other people, but write up your own answers, don't just copy from someone else.
- This exam has 7 problems. There are **340 points total**.

Good luck!

60 pts.

Problem 1. Use the method of undetermined coefficents to find the general solution. You have to use undetermined coefficents, no credit for any other method (of course, you could check yourself). Remember the Modification Rule!

A.

$$y'' - y' - 2y = xe^x.$$

B.

$$y'' - y' - 2y = xe^{-x}.$$

C.

$$y'' + 4y = \sin(2x)$$

---

40 pts.

Problem 2. In each part, find the form of the partial fraction decomposition of the given function, showing the undetermined coefficients. **Do not determine the coefficients!** If you're doing long computations, you're doing it wrong.

A.

$$\frac{s^4 + 6}{(s-1)(s-2)(s-3)^3}$$

B.

$$\frac{1}{s^3(s^2+4)^2}$$

---

60 pts.

Problem 3. In each part, find the inverse Laplace transform.

A. In this part, find the partial factions decomposition by hand.

$$\frac{7s^2 - 11s - 5}{(s-1)^2(s+2)}$$

B. In this part, use a calculator to find the partial fractions decomposition.

$$\frac{4s^6 - 14s^5 + 20s^4 - 21s^3 + 44s^2 + 3s + 8}{(s^2+1)^2(s-2)^2(s+1)}$$

C. Hint: complete the square in the denominator.

$$\frac{s+1}{s^2 + 6s + 34}$$

---

40 pts.

Problem 4. Solve the following initial value problems **by the method of Laplace Transforms**.

A.

$$y'' - 3y' + 2y = \sin(t), \qquad y(0) = 1, \quad y'(0) = -1$$

B.

$$y'' - 3y' + 2y = te^{2t}, \qquad y(0) = 2, \quad y'(0) = 1$$

---

40 pts.

Problem 5. Find the Laplace Transform of the function

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ t^2, & 1 < t < 2 \\ t + 1, & 2 < t < \infty. \end{cases}$$

---

40 pts.

Problem 6. Find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^3} + e^{-s}\frac{s}{(s^2+4)(s-2)} - e^{-2s}\frac{s}{(s-1)(s+2)}$$

---

60 pts.

Problem 7. Use Laplace Transforms to solve the differential equation

$$y'' + y = \mathscr{U}(t-1)t^2, \qquad y(0) = 0, \quad y'(0) = 1.$$
