EXAM

Exam 3, Takehome Exam

Math 3350–D01, Spring 2013

November 15, 2013

- This is a Take-home exam.
- Write all of your answers on separate sheets of paper. You can keep the exam questions. You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- Unless otherwise instructed, you can use a calculator to do integrals and partial fractions decompositions. State clearly what you are going to put into the calculator and what you got out.
- You can use the textbook and your notes. You can discuss the problems with other people, but write up your own answers, don't just copy from someone else.
- This exam has 9 problems. There are **420 points total**.

Good luck!

40 pts.

Problem 1. In each part, find the form of the partial fraction decomposition of the given function, showing the undetermined coefficients. **Do not determine the coefficients!** If you're doing long computations, you're doing it wrong.

A.

$$\frac{s^4+6}{(s-1)(s-2)(s-3)^3}$$

B.

$$\frac{1}{s^3(s^2+4)^2}$$

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60 pts.

Problem 2. In each part, find the inverse Laplace transform.

A. In this part, find the partial factions decomposition by hand.

$$\frac{7s^2-11s-5}{(s-1)^2(s+2)}$$

B. In this part, use a calculator to find the partial fractions decomposition.

$$\frac{4s^6-14s^5+20s^4-21s^3+44s^2+3s+8}{(s^2+1)^2(s-2)^2(s+1)}$$

C. Hint: complete the square in the denominator.

$$\frac{s+1}{s^2+6s+34}$$

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40 pts.

Problem 3. Solve the following initial value problems **by the method of Laplace Transforms**.

A.

$$y''-3y'+2y=\sin(t), \qquad y(0)=1, \quad y'(0)=-1$$

B.

$$y''-3y'+2y=te^{2t}, \qquad y(0)=2, \quad y'(0)=1$$

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40 pts.

Problem 4. Find the Laplace Transform of the function

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ t^2, & 1 < t < 2 \\ t+1, & 2 < t < \infty. \end{cases}$$

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40 pts.

Problem 5. Find the inverse Laplace transform of the function

$$F(s) = \frac{1}{s^3} + e^{-s}\frac{s}{(s^2+4)(s-2)} - e^{-2s}\frac{s}{(s-1)(s+2)}$$

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60 pts.

Problem 6. Use Laplace Transforms to solve the differential equation

$$y'' + y = \mathscr{U}(t-1)t^2, \qquad y(0) = 0, \quad y'(0) = 1.$$

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40 pts.

Problem 7. Find the following convolutions directly from the definition. Compute the integrals by hand.

A. $e^{-2t} * e^{5t}$.

B. $t * t^3$.

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40 pts.

Problem 8. **Use Laplace Transforms** to find the convolution

$$t^2 * \cos(2t).$$

Use a calculator to do the partial fractions!

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60 pts.

Problem 9.

A. Use Laplace Transforms to solve the differential equation

$$y'' + y = \delta(t-1), \qquad y(0) = 1, \quad y'(0) = 1.$$

B. Without any further computation, what is the solution of

$$y'' + y = 0, \qquad y(0) = 1, \quad y'(0) = 1?$$

C. Without further computation, what is the solution of

$$y'' + y = \delta(t-1), \qquad y(0) = 0, \quad y'(0) = 0?$$
