EXAM

Exam 1, Makeup version 1

Math 3350, Summer II, 2013

Original in class on June17, 2016

- Write all of your answers on separate sheets of paper. Do not write on the exam handout. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **400 points total**.

Good luck!

50 pts.

Problem 1. Consider the autonomous differential equation

$$\frac{dy}{dx} = y(y-2)(y+3).$$

A. Find the constant solutions of this equation. Draw the phase portrait for this equation. Classify each equilibrium point as stable, unstable or semi-stable.

B. In the xy-plane, sketch the graphs of typical solutions in the regions divided by the constant solutions.

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140 pts.

Problem 2. In each part, find the general solution of the differential equation, or solve the given initial value problem. You must show the steps in solving the equation by one of the methods given in class, you can't just write down the answer.

A.
$$\frac{dy}{dx} + \frac{2xy}{x^2+1} = x$$

B.
$$\frac{dy}{dx} = 2xy^4$$

C.
$$\frac{dy}{dx} = 3x^2y$$

D.
$$\frac{dy}{dx} + 5y = -\sin(x)e^{-5x}, \qquad y(0) = 4.$$

E.
$$\frac{dy}{dx} + 2y = e^{2x}y^{-2}$$

F.
$$\frac{dy}{dx} = \frac{x^2+2y^2}{xy}$$

G.
$$\frac{dy}{dx} = (x+y+3)^2$$

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40 pts.

Problem 3. The following equation is exact. Solve it.

$$(2xy^2 + 8x^3y^3 + 3x^2)\,dx + (2x^2y + 6x^4y^2 + 1)\,dy = 0.$$

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40 pts.

Problem 4. The following differential equation is **not** exact. Find an integrating factor that is a function of x alone, or an integrating factor that is a function of y alone. Find the general solution of the differential equation.

$$2xy^3\,dx + (3x^2y^2 - x^2y^3)\,dy = 0$$

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60 pts.

Problem 5. A can of soda at a temperture of 35° F is brought into a room. The temperature of the room is 70° F. After 5 min the temperature of the can is 45° F.

Recall that the differential equation for newton's law of cooling is

$$\frac{dT}{dt} = k(T - T_M).$$

A. Write down the solution of the differential equation. You don't have to show the procedure, just write down the answer.

B. Find the value of k for this problem. Give an exact answer, not an approximation.

C. Find the temperature of the can after 10 minutes. Give an exact answer and an approximation to two decimal places.

D. At what time will the temperature of the can be 69 degrees? Give an exact answer and an approximation to two decimal places.

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70 pts.

Problem 6. A tank contains 100 gallons of water. Five gallons of brine per minute flow into the tank, each gallon of brine containing 1 pound of salt. Five gallons of brine flow out of the tank per minute. Assume that the tank is kept well stirred.

Find a differential equation for the number of pounds of salt in the tank (call it y, say).

Assuming the tank intially contains 1 pound of salt, solve this differential equation.

How much salt is in the tank after 5 minutes? Give an exact solution and a numerical answer accurate to two decimal places

At what time will there be 95 lbs of salt in the tank? Give an exact answer and a numerical answer accurate to two decimal places.
