EXAM

Exam 1

Math 3350–202, Summer II, 2013

July 22, 2013

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 7 problems. There are **390 points total**.

Good luck!

50 pts.

Problem 1. Consider the autonomous differential equation

$$\frac{dy}{dx} = y(y-2)(y+3).$$

A. Find the constant solutions of this equation. Draw the phase portrait for this equation. Classify each equilibrium point as stable, unstable or semi-stable.

B. In the xy-plane, sketch the graphs of typical solutions in the regions divided by the constant solutions.

---

110 pts.

Problem 2. In each part, find the general solution of the differential equation, or solve the given initial value problem. You must show the steps in solving the equation by one of the methods given in class, you can't just write down the answer.

A.

$$\frac{dy}{dx} = 3x^2y^4$$

B.

$$\frac{dy}{dx} = 2x(1+y)$$

C.

$$\frac{dy}{dx} + \frac{3}{x}y = x^5, \qquad y(1) = 3$$

D.

$$\frac{dy}{dx} + 2y = e^{3x}y^2$$

E.

$$\frac{dy}{dx} = -1 + (2x+y)^2$$

---

40 pts.

Problem 3. The following equation is exact. Solve it.

$$(3x^2y^2 + 2xy^4 + 2x)\,dx + (2x^3y + 4x^2y^3 + 1)\,dy = 0.$$

---

40 pts.

Problem 4. The following differential equation is **not** exact. Find an integrating factor that is a function of x alone, or an integrating factor that is a function of y alone. Find the general solution of the differential equation.

$$2xy^3\,dx + (3x^2y^2 - x^2y^3)\,dy = 0$$

---

40 pts.

Problem 5. The following differential equation is homogenous. Solve it by an appropriate substitution.

Find the general solution of the differential equation.

$$(x^2 + xy + y^2)\,dx - x^2\,dy = 0$$

---

70 pts.

Problem 6. A cup of coffee at a temperture of 190° F is brought into a room. The temperature of the room is 70° F. After 10 min the temperature of the coffee is 170° F.

Recall that the differential equation for newton's law of cooling is

$$\frac{dT}{dt} = k(T - T_M).$$

Start by writing down the solution of this equation (you don't need to show the details of solving the equation).

A. Find the value of k for this problem. Give an exact answer, not an approximation.

B. Find the temperature of the coffee after 20 minutes. Give an exact answer and an approximation to two decimal places.

C. At what time will the temperature of the coffee be 71 degrees? Give an exact answer and an approximation to two decimal places.

---

40 pts.

Problem 7. A tank contains 200 gallons of water. Five gallons of brine per minute flow into the tank, each gallon of brine containing 1 pound of salt. Five gallons of water flow out of the tank per minute. Assume that the tank is kept well stirred.

Find a differential equation for the number of pounds of salt in the tank (call it y, say).

Assuming the tank intially contains no salt, solve this differential equation.

At what time will there be 100 lbs of salt in the tank? Give an exact answer and a numerical answer accurate to two decimal places.
