Problem Set

Practice Problems

Math 2360–D01, Fall 2017

November 1, 2017

- Practice problems for Exam 2. Solutions will be posted.
- You **must** show enough work to justify your answers.' Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

Good luck!

70 pts.

Problem 1. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & -3 & 0 & 1 \\ 1 & 0 & -1 & 0 & -2 \\ 1 & 1 & -3 & 1 & 6 \\ 1 & 0 & -1 & 1 & 3 \\ -3 & 0 & 3 & -1 & 1 \end{bmatrix}$$

The RREF of A is the matrix

$$R = \begin{bmatrix} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A. Find a basis for the nullspace of A.

B. Find a basis for the rowspace of A.

C. Find a basis for the columnspace of A.

D. What is the rank of A?

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80 pts.

Problem 2. Consider the vectors

$$v_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -2 \\ -5 \\ -3 \\ 0 \\ -3 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad v_5 = \begin{bmatrix} -2 \\ 7 \\ 6 \\ -7 \\ 1 \end{bmatrix}.$$

Let $S \subseteq \mathbb{R}^5$ be defined by

$$S = \operatorname{span}(v_1, v_2, v_3, v_4, v_5).$$

A. Find a basis for S. What is the dimension of S?

B. For each of the vectors v_1, v_2, v_3, v_4, v_5 which is not in the basis, express that vector as linear combination of the basis vectors.

C. Consider the vectors

$$w_1 = \begin{bmatrix} 9 \\ 6 \\ 4 \\ 1 \\ 8 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 2 \\ 2 \end{bmatrix}.$$

Determine if each of these vectors is in S. If the vector is in S, write it as a linear combination of the basis vectors for S you found in the first part.

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40 pts.

Problem 3. In each part, determine if the given vectors in $\mathbb{R}^4$ are linearly independent. If they are dependent, find constants c_1, c_2 and c_3, not all zero, so that

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0.$$

A.

$$v_1 = \begin{bmatrix} -4 \\ 4 \\ -1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 10 \\ -2 \\ -1 \\ -1 \end{bmatrix}$$

B.

$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 7 \\ 13 \\ 21 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 26 \end{bmatrix}$$

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40 pts.

Problem 4. In each case, determine if the given vectors span $\mathbb{R}^3$ and, if so, pare the given list of vectors down to a basis of $\mathbb{R}^3$.

A.

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

B.

$$v_1 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ 8 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}.$$

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40 pts.

Problem 5. The following list of vectors is linearly independent in $\mathbb{R}^4$:

$$v1 = \begin{bmatrix} 2 \\ 7 \\ 6 \\ -3 \end{bmatrix}, \quad v2 = \begin{bmatrix} 9 \\ 32 \\ 26 \\ -13 \end{bmatrix}, \quad v3 = \begin{bmatrix} -1 \\ -3 \\ -2 \\ 1 \end{bmatrix}$$

Complete this list to a basis of $\mathbb{R}^4$ by adding some of the standard basis vectors.
