EXAM

Exam 1, Version 1

Math 2360-D01, Fall 2019

Sept. 30ff, 2016

- Write all of your answers on separate sheets of paper. Do not write on the exam handout. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **265 points total**.

Good luck!

50 pts.

Problem 1. In each part, determine if the matrix multiplication is defined. If it is undefined, say "undefined," otherwise give the result of the operation.

A.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$$

B.

$$\begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

C.

$$\begin{bmatrix} -1 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

D.

$$\begin{bmatrix} -1 & 2 & 1 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

E.

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 3 \\ 5 & 6 \end{bmatrix}$$

---

50 pts.

Problem 2. In each part, you are given the augmented matrix of a linear system. The coefficient matrix is already in Reduced Row Echelon Form. Determine if the system is consistent, and if it is consistent find all solutions.

(a.)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(b.)

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{array}\right]$$

(c.)

$$\left[\begin{array}{ccccc|c} 1 & -1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

---

40 pts.

Problem 3. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in row echelon form. (Not *reduced* row echelon form!) Determine if the system is consistent. If so, use **back substitution** to find all the solutions of the system.

A.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

B.

$$\left[\begin{array}{ccccc|c} 1 & -2 & 3 & 1 & 1 & 2 \\ 0 & 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

---

40 pts. **Problem 4**. In each part solve the linear system using the Gauss-Jordan method (i.e., reduce the coefficent matrix to Reduced Row Echelon Form). Show the augmented matrix you start with and the augmented matrix you finish with. It's not necessary to show individual row operations, you can just hit the RREF key on your calculator.

A.

$$\begin{aligned} x_1 - 2x_3 &= 3 \\ 8x_1 + x_2 - 13x_3 + 3x_4 &= 41 \\ 2x_1 - 4x_3 + x_4 &= 11 \end{aligned}$$

B.

$$\begin{aligned} 6x_1 + 7x_2 + 5_3 &= 15 \\ 2x_1 + x_2 + 3x_3 &= 1 \\ x_1 + x_3 &= 1 \end{aligned}$$

---

40 pts. **Problem 5**. In each part, **Use row operations** to determine if the matrix A is invertible and, if so, to find the inverse. Show the indivdual row operations, one by one. You can use a calculator to do the row operations if you wish. Give the matrix entries in fractional form. Be sure to label your answer for the inverse. Sorry, no credit for using a different method!

A.

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}.$$

B.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

---

45 pts.

Problem 6. Consider row operations on matrices with 3 rows.

Recall that for each row operation there is a corresponding elementary matrix E so that EA is the same as the matrix obtained by applying the row operation to A.

A. Consider the row operation $R_2 \leftrightarrow R_3$.

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation.Find the inverse row operation (in the notation in which the row operations are given in this problem).

   iii.) Find the elementary matrix that corresponds to the inverse row operation.

B. Consider the row operation $R_3 \leftarrow 3R_3$.

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation (in the notation in which the row operations are given in this problem).

   iii.) Find the elementary matrix that corresponds to the inverse row operation.

C. Consider the row operation $R_2 \leftarrow R_2 - 3R_1$

   i.) Find the corresponding elementary matrix E.

   ii.) Find the inverse row operation (in the notation in which the row operations are given in this problem).

   iii.) Find the elementary matrix that corresponds to the inverse row operation.
