EXAM

Exam 1

Math 2360–D01, Spring 2015

Feburary 17, 2015

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 6 problems. There are **270 points total**.

Good luck!

50 pts.

Problem 1. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in reduced row echelon form. Determine if the system is consistent and, if it is consistent, find all solutions.

A.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

B.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

C.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & 4 & 2 \\ 0 & 1 & 0 & -2 & -1 & 1 \\ 0 & 0 & 1 & 3 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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40 pts.

Problem 2. In each part you are given the augmented matrix of a system of linear equations, with the coefficient matrix in row echelon form. (Not *reduced* row echelon form!) Use **back substitution** to find all the solutions of the system.

A.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

B.

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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40 pts.

Problem 3. Solve the linear system below. Use your calculator to find the RREF, but write down the augmented matrix and the matrix you wind up with, and then find all solutions.

$$\begin{aligned} 2x_1 + 2x_2 - 2x_3 &= 8 \\ x_2 + x_3 &= 1 \\ 2x_1 + x_2 - 3x_3 &= 7 \end{aligned}$$

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40 pts.

Problem 4. **Use row operations** to determine if the matrix A is invertible and, if so, to find the inverse. Show the individual row operations, one by one, stating what row operation you are using. You can use a calculator to do the row operations if you wish. Give the matrix entries in **fractional form**.

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$$

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40 pts.

Problem 5. In each part, you are given a square matrix. Determine if the matrix is invertible and, if so, find the inverse. Use the Gauss-Jordan method. You can just use the `rref` key on the calculator, but show the matrix you put in to the calculator, the matrix you get out of the calculator, and give your interpretation of the result.

A.

$$A = \begin{bmatrix} 0 & 3 & 1 & 17 \\ 0 & 15 & 5 & 86 \\ 1 & -9 & -3 & -36 \\ -1 & 8 & 3 & 32 \end{bmatrix}$$

B.

$$A = \begin{bmatrix} -4 & 4 & 8 & 0 \\ -7 & 5 & 16 & -4 \\ -21 & 22 & 41 & 2 \\ -2 & -1 & 7 & -6 \end{bmatrix}$$

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60 pts.

Problem 6. In each part, find the elementary matrix E so that $EA = B$. Also find E^{-1}.

The matrix A is

$$A = \begin{bmatrix} 3 & 1 & 2 & 4 \\ 0 & 5 & 4 & 1 \\ 5 & 7 & 1 & 6 \end{bmatrix}$$

A.

$$B = \begin{bmatrix} 3 & 1 & 2 & 4 \\ 0 & -10 & -8 & -2 \\ 5 & 7 & 1 & 6 \end{bmatrix},$$

B.

$$B = \begin{bmatrix} 3 & 1 & 2 & 4 \\ 5 & 7 & 1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix},$$

C.

$$B = \begin{bmatrix} -7 & -13 & 0 & -8 \\ 0 & 5 & 4 & 1 \\ 5 & 7 & 1 & 6 \end{bmatrix},$$
