40 pts.

Problem 1. The acceleration vector of a particle moving in three dimensional space is given by

$$\mathbf{A}(t) = t\mathbf{i} + e^{2t}\mathbf{j} + \sin(t)\mathbf{k}.$$

The initial velocity of the particle is

$$\mathbf{V}(0) = \mathbf{i} + \frac{3}{2}\mathbf{j} + \mathbf{k},$$

and the initial postion is

$$\mathbf{R}(0) = 2\mathbf{i} + \frac{1}{2}\mathbf{j} + 3\mathbf{k}.$$

Find the velocity $\mathbf{V}(t)$ and the position $\mathbf{R}(t)$ of the particle for all t.

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40 pts.

Problem 2. A cannon has a muzzle speed of 800 feet per second.

A. What is the maximum range of this cannon? Give a numerical answer accurate to two decimal places.

B. What angles can the cannon be fired at to hit a target $15,000$ feet away? Give a numerical answers in degrees, accurate to two decimal places.

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50 pts.

Problem 3. Consider the function

$$f(x, y) = \sin(x^2 y) + x + y^3.$$

Find the partial derivatives f_x, f_y, f_{xx}, f_{xy}, f_{yx} and f_{yy}.

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40 pts.

Problem 4. Let z be defined implicitly as a function of x and y by the equation

$$xy\sin(yz^2) = 1.$$

Use implicit differentiation to find the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$.

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50 pts.

Problem 5. A rectangle is measured to have a length of 5 feet and a width of 3 feet. The maximum error possible error in these measurements is 1/2 inch (careful about the units!). **Use increments** to estimate the maximum possible error in the calculated volume. What is the maximum possible percentage error in the calculated volume? Give numerical answers accurate to two decimal places.

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40 pts.

Problem 6.

Let $z = f(x, y) = x^2y^2 + xy$, where $x = 2t$ and $y = t^2$. Find dz/dt in the following two ways.

A. Express z explicitly as a function of t and differentiate.

B. Use the Chain Rule for partial derivatives, expressing your final answer as a function of t.

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40 pts.

Problem 7. Consider the function

$$f(x, y) = x^2y + x + y^2$$

A. Find the directional derivative of f at the point $P(2, 1)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.

B. In what direction should you go from P to get the greatest rate of change in f? What is this maximum rate of change?

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40 pts.

Problem 8. Find the critical points of the function

$$f(x, y) = x^2 + y^3 - 6xy.$$

Classify each of the critical points as a relative maximum, relative minimum, or a saddle point.

EXAM

Exam 1

Math 2350-02, Summer II 2008

July 22, 2008

- Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.
- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This exam has 8 problems. There are **340 points total**.

Good luck!