Problem 1. In each part, find the general solution of the differential equation.

A. \[ \frac{dy}{dx} = x\sqrt{y}. \]

B. \[ \frac{dy}{dx} + \frac{3}{x}y = x^5. \]

C. \[ \frac{dy}{dx} + 3y = e^{x^3}. \]

D. \[ \frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}. \]

Problem 2. The following equation is exact. Solve it.

\[(2xy + y^2)\,dx + (x^2 + 2xy + 3y^2)\,dy = 0\]

Problem 3. Find an integrating factor and use it to solve the equation

\[y\,dx + (x + xy)\,dy = 0.\]

Problem 4. Solve the differential equation

\[(x - 1)\,dx + (x + y + 1)\,dy = 0\]

by the substitution \(x = u + h\) and \(y = v + k\), where \(h\) and \(k\) are constants to be determined.

Problem 5. A projectile with a mass of 1 slug (i.e., the weight is \(mg = 32\) pounds) is fired straight up from ground level at a velocity of 200 feet per second. The force of air resistance on the projectile is \(-2v\), where \(v\) is the velocity of the projectile.

How long does it take for the projectile to come to rest at the top of its trajectory? How high does it go? [Give numerical answers accurate to two decimal places.]
• Write all of your answers on separate sheets of paper. You can keep the exam questions when you leave. You may leave when finished.

• You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This exam has 5 problems. There are **200 points total**.

    Good luck!