This is a take-home exam, due by 5 p.m., Monday, May 2.

Write all of your answers on separate sheets of paper. You can keep the exam questions.

You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

This exam has 9 problems. There are **360 points total**.

Good luck!
Instructions: You can work together, but after figuring out how to do the problem, write up your own solution. I should not be seeing identical text.

Unless otherwise instructed, you can use a symbolic calculator to do the integrals. However, do one one-dimensional integral at a time and indicate clearly where you have used the calculator.

Problem 1. Use Lagrange Multipliers to find the absolute maximum and minimum of the function \( f(x, y, z) = xy + z \) on the spherical surface \( x^2 + y^2 + z^2 = 9 \).

Problem 2. Let \( R \) be the region in the \( xy \)-plane bounded by \( y = x^2 \) and \( y = 2x \). Find the integral
\[
\int \int_R x \, dx \, dy.
\]
In this problem, work the integrals by hand and show the steps.

Problem 3. In each part, sketch the region of integration and find an equivalent iterated integral with the order of integration reversed.

A.
\[
\int_0^1 \int_{\sqrt{x}}^1 f(x, y) \, dy \, dx.
\]

B.
\[
\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x, y) \, dx \, dy.
\]

Problem 4. Let \( R \) be the region in the first quadrant enclosed by one leaf of the four leaf rose \( r = \sin(2\theta) \). Find \( \bar{x} \), the \( x \)-coordinate of the centroid of \( R \). What is \( \bar{y} \)?
Problem 5. Let $D$ be the region that in three-dimensional space bounded below by $z = 0$, above by $z = y$ and laterally by the plane $y = 1$ and the cylinder $y = x^2$.

A. Find an iterated integral for finding the volume of $D$ where the first integration is with respect to $z$.

B. Find an iterated integral for finding the volume of $D$ where the first integration is with respect to $x$.

C. Find an iterated integral for finding the volume of $D$ where the first integration is with respect to $y$.

D. Evaluate one of these integrals by hand.

Problem 6. Let $D$ be the region in three-dimensional space bounded above by the paraboloid $z = 6 - x^2 - y^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. Find the volume of $D$, and the moment of inertia for rotation about the $z$-axis.

Problem 7. Let $D$ be the region in three-dimensional space bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. Use spherical coordinates to find the volume of $D$, $\bar{z}$ (the $z$-coordinate of the centroid) and the moment of inertia for rotation about the $z$-axis.

Problem 8. Let $D$ be the spherical cap bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the plane $z = h$, where $0 < h < a$. Use a triple integral to find the volume of $D$.

Problem 9. Let $R$ be the region in the $xy$-plane bounded below by the $x$-axis and above by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$ 

Find $\bar{y}$, the $y$-coordinate of the centroid of $R$.

Start by making the change of variables $x = au$, $y = bv$. 

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