EXAM

Take-home Exam
Exam #3

Math 3350-01, Second Summer Session 2004

July 30, 2004

• Write all of your answers on separate sheets of paper. You can keep the exam questions.

• You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This exam has 7 problems. There are 240 points total.

• You may discuss the exam with other students, but write up the answers by yourself.

• Due Monday, August 2.

Good luck!
Problem 1. In each part, find the Inverse Laplace Transform.

A. In this part, work out the partial fractions decomposition by hand,

\[ \frac{1}{s^2(s-1)}. \]

B. In this part, you can do the partial fractions decomposition by machine,

\[ \frac{3s^3 - 2s^2 + 2s + 2}{(s^2 + 4)(s-1)^2}. \]

Problem 2. Find the Laplace transform of the function

\[ f(t) = \begin{cases} \frac{t}{s}, & 0 \leq t < 1, \\ \frac{t^2}{s}, & 1 \leq t < 2 \\ 0, & 2 < t < \infty \end{cases} \]

Problem 3. Find the Inverse Laplace Transform of the function

\[ \frac{1}{s-1} + e^{-s} \frac{1}{s^2+4} + e^{-2s} \frac{1}{(s+2)^2}. \]

Problem 4. In each part, solve the initial value problem using Laplace transforms. (No credit for using a different method.)

A. 

\[ y'' - 4y' + 4y = te^{2t}, \quad y(0) = 1, \quad y'(0) = 0. \]

B. 

\[ y'' + 4y = \delta(t-1) + tu(t-2), \quad y(0) = 0, \quad y'(0) = 1. \]
Problem 5. In the following problems, use formulas (1) and (6) from section 5.4.

A. Find the Laplace transform of \( f(t) = t^2 \cos(2t) \).

B. Find the Laplace transform of
\[
f(t) = \frac{e^{2t} - e^t}{t}.
\]
You may assume the fact that \( \lim_{t \to 0^+} f(t) \) exists.

C. Find the inverse Laplace transform of the function
\[
G(s) = \frac{1}{2} \ln\left(\frac{s^2}{s^2 + 1}\right).
\]

Problem 6. Find the convolution \( t \ast t^3 \) directly from the definition.

Problem 7. Find the convolution \( \cos(2t) \ast \sin(3t) \) using Laplace transforms.