
PROBLEM SET

Practice Problems for Exam #2

Math 2350, Fall 2004

Nov. 7, 2004

Corrected Nov. 10

- You **must** show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).
- This problem set has 12 problems.

Good luck!

Problem 1. Consider the function $f(x, y) = xy^2 \sin(x^2y)$. Find the partial derivatives $f_x, f_y, f_{xx}, f_{xy}, f_{yx}$ and f_{yy} .

Problem 2. Use the chain rule for partial derivatives to find $\partial r/\partial x$ and $\partial r/\partial y$ where $r = uv - u^2 - v^2$, $u = x^2y$ and $v = 2x - y$. Express your answer in terms of x and y .

Problem 3. Suppose that $p = f(u, v, w)$, $u = g(x, y)$, $v = h(y, z)$ and $w = k(x, y, z)$. Write the chain rule formals for the partial derivatives of p with respect to x, y and z .

Problem 4. A box is measured to have a length of 15 inches, a width of 10 inches and a depth of 5 inches. From these measurements, we would calculate the volume to be 750 cubic inches. If there is a possible error of up to 0.25 inch in the measurements, use differentials to estimate the maximum possible error in the calculated volume.

Problem 5. A rectangular block of ice is melting in the sun. At a certain instant the block has a height of 5 inches, a length of 10 inches, and a width of 12 inches, and each of the dimensions is decreasing at a rate of 2 inches per hour. How fast is the volume of the block changing?

Problem 6. Find the directional derivative of the function $f(x, y) = x^2 + 2xy$ at the point $(1, 2)$ in the direction of the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.

Problem 7. Suppose that the temperature at a point (x, y) in the plane is given by $f(x, y) = x^2 + xy + y^2$. A bug is walking around in the plane with a speed of 5 units per minute.

A. At a certain instant the bug is at the point $(2, 1)$ and has velocity vector $\mathbf{v} = (3, -4)$. What rate of change of temperature is the bug experiencing at this instant?

B. Suppose the bug is at $(2, 1)$. If the bug wants to cool off, what direction should he travel (at speed 5) to get fastest decrease in temperature? What is this rate of change of temperature?

Problem 8. Find the equation of the tangent plane to the cone $z^2 = x^2 + y^2$ at the point $P = (4, -3, 5)$. Find parametric equations for the normal line to the surface at this point.

Problem 9. Locate and classify the critical points of the function

$$f(x, y) = x^4 + 4xy + y^4.$$

Problem 10. Find the absolute max and min of the function $f(x, y) = xy$ on the disk $x^2 + y^2 \leq 2$. You can parametrize the boundary, or use Lagrange multipliers.

Problem 11. Find the absolute max and min of the function $f(x, y, z) = xyz^2$ on the spherical surface $x^2 + y^2 + z^2 = 128$. (The absolute max and min exist because the surface is closed and bounded.)

Problem 12. Consider the curve C formed by the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. Find the points on the curve that are closest and farthest from the origin. (The curve is closed and bounded.)
