PROBLEM SET

Practice Problems for Exam #1

Math 1352, Fall 2004

Oct. 1, 2004

• You must show enough work to justify your answers. Unless otherwise instructed, give exact answers, not approximations (e.g., $\sqrt{2}$, not 1.414).

• This problem set has 9 problems.

Good luck!
Problem 1. Let $R$ be the region bounded by the curves $x = y^2$ and $y = x$.

A. Find the volume of the solid generated by revolving the region $R$ around the $x$-axis.

B. Find the volume of the solid generated by revolving the region $R$ around the $y$-axis.

Problem 2.

The base of a solid is the region in the $xy$-plane bounded by the lines $y = 2x$ and $x = 1$. The cross sections of the solid perpendicular to the $y$-axis are squares. Find the volume of the solid.

Problem 3. Find the area of one leaf of the four-leaf rose whose polar equation is $r = \sin(2\theta)$.

Problem 4.

Find the area of the region that is inside the cardioid $r = 1 + \cos(\theta)$ and outside the circle $r = 1$.

Problem 5.

Find the arc length of the graph of

\[ f(x) = \frac{1}{3}x^3 + \frac{1}{4}x^{-1} \]

on the interval from $[1, 2]$. (This is Problem 9 on page 393 of the book.)
Problem 6. A tank has the shape of a hemisphere (see picture) of radius 1 meter. The tank is full of water, which weights 9800 N/m$^3$. How much work is required to pump all of the water to a point one meter above the top of the tank?

![Hemisphere Diagram]

Problem 7.
The vertical cross sections of a tank are isosceles triangles, point upwards. The bottom of the tank is 4 feet across and it is 8 feet high. If the tank is filled with water to a depth of 4 feet, what is the total force on one end of the tank? (The weight density of water is $w = 62.4 \text{ lbs/ft}^3$.)

Problem 8. Let $R$ be the region bounded by the curves $y = 1 - x^2$ and $y = 1 - x$.

A. Find the area of $R$.

B. Find the $y$-coordinate of the centroid of $R$.

C. Use the Theorem of Pappus to find the volume of the solid generated when the region $R$ is revolved around the $x$-axis.
Problem 9.
Find the following integrals.

A. \[
\int x \cos(2x) \, dx.
\]

B. \[
\int x^2 \ln(x) \, dx.
\]