EXAM

Exam #1
Math 1430, Fall 2001
September 21, 2001

ANSWERS
Problem 1. Preform the indicated operations and simplify.

A. \[3(x^2 + 4x - 7) - 2(x^2 - 9x + 5)\]

Answer:
\[3(x^2 + 4x - 7) - 2(x^2 - 9x + 5) = 3x^2 + 12x - 21 - 2x^2 + 18x - 10 = x^2 + 30x - 31.\]

B. \[(2x - 1)(5x + 3)\]

Answer:
\[(2x - 1)(5x + 3) = (2x)(5x) + (2x)(3) + (-1)(5x) + (-1)(3) \quad \text{(FOIL)}\]
\[= 10x^2 + 6x - 5x - 3 = 10x^2 + x - 3.\]

Problem 2. If possible, factor the polynomial \(2x^2 - 4x - 6\) using integer coefficients.

Answer:
Comparing with \(ax^2 + bx + c\) we see that \(a = 2\), \(b = -4\) and \(c = -6\). Thus, \(ac = -12\). We want to factor \(-12\) as \(pq\), with \(p + q = b = -4\). This can be done with \(p = -6\) and \(q = 2\). Then we have
\[2x^2 - 4x - 6 = 2x^2 + (2 - 6)x - 6 = 2x^2 + 2x - 6x - 6 = 2x(x + 1) - 6(x + 1) = (2x - 6)(x + 1) = 2(x - 3)(x + 1).\]

Problem 3. In each part, perform the indicated operation and reduce to lowest terms.

A. \[
\frac{1}{x} - \frac{1}{x+1}
\]
Answer:
\[
\frac{1}{x} - \frac{1}{x+1} = \frac{1(x+1)}{x(x+1)} - \frac{1(x)}{(x+1)(x)} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} = \frac{x+1-x}{x(x+1)} = \frac{1}{x(x+1)}
\]

B.
\[
\frac{x}{x^2 - 4} = \frac{x}{x^2} \cdot \frac{1}{x+2}
\]

Answer:
\[
\frac{x}{x^2 - 4} = \frac{x}{x^2 - 4} \cdot \frac{x + 2}{x^2} = \frac{x}{(x-2)(x+2)} \cdot \frac{x + 2}{x^2} = \frac{1}{x(x-2)}.
\]

**Problem 4.** In each part, simplify and express answers using positive exponents only.

A.
\[
\frac{5x^{-2}y^{-3}}{10xy^{-5}}
\]

Answer:
\[
\frac{5x^{-2}y^{-3}}{10xy^{-5}} = \frac{5}{10} x^{-2-1} y^{-3-(-5)} = \frac{1}{2} x^{-3} y^2 = \frac{y^2}{2x^3}.
\]
B. 

$$\left(2x^{-2}y^4\right)^{-3}$$

Answer:

$$\left(2x^{-2}y^4\right)^{-3} = 2^{-3}(x^{-2})^{-3}(y^4)^{-3}$$

$$= 2^{-3}x(-2)(-3)y^4(-3)$$

$$= 2^{-3}x^6y^{-12}$$

$$= \frac{x^6}{8y^{12}}.$$
Problem 6. Solve the inequality $0 < 4 - 2x \leq 2$. Graph the solution set.

Answer:

We start with the inequality

$$0 < 4 - 2x \leq 2.$$ 

Adding $-4$ to each expression gives

$$-4 < -2x \leq -2.$$ 

Now multiply through the inequality by $-1/2$. **Remember that multiplying through by a negative number reverse the inequalities.** Thus, we get

$$(-1/2)(-4) > (-1/2)(-2)x \geq (-1/2)(-2)$$

which simplifies to

$$2 > x \geq 1,$$

which is the same as

$$1 \leq x < 2$$

or, in interval notation, $[1,2)$. The graph of the solution set is shown in Figure 1.

Problem 7.

A. Solve $2x^2 - 6x = 0$.

Answer:

$$2x^2 - 6x = 2x(x - 3),$$

so the solutions are $x = 0$ and $x = 3$.  

Figure 1: Graph of the Solution Set for Problem 6
B. Solve $2x^2 - x - 1 = 0$.

**Answer:**
Comparing with $ax^2 + bx + c$ we have $a = 2$, $b = -1$ and $c = -1$. We want to find $p$ and $q$ so that $ac = -2 = pq$ and $b = -1 = p + q$. We can take $p = -2$, and $q = 1$. Then

$$2x^2 - x - 1 = 2x^2 + (1 - 2)x - 1$$
$$= 2x^2 + x - 2x - 1$$
$$= x(2x + 1) - (2x + 1)$$
$$= (2x + 1)(x - 1).$$

The factor $x - 1$ is zero when $x = 1$. The factor $2x + 1 = 0$ when $2x = -1$ or $x = -1/2$. Thus the solutions are $x = 1$ and $x = -1/2$.

C. **Use the quadratic formula to solve** $4x^2 + 11x - 3 = 0$. Give a factorization of this polynomial with integer coefficients, using the factor theorem.

**Answer:**
Comparing $4x^2 + 11x - 3$ to $ax^2 + bx + c$, we have $a = 4$, $b = 11$ and $c = -3$. By the quadratic formula, the roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-11 \pm \sqrt{11^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{-11 \pm \sqrt{121 + 48}}{8}$$
$$= \frac{-11 \pm \sqrt{169}}{8}$$
$$= \frac{-11 \pm 13}{8}.$$ 

Thus, the two roots of $4x^2 + 11x - 3 = 0$ are

$$r_1 = \frac{-11 + 13}{8} = 2/8 = \frac{1}{4}$$
$$r_2 = \frac{-11 - 13}{8} = -24/8 = -3.$$ 

According to the factor theorem, we have

$$4x^2 + 11x - 3 = a(x - r_1)(x - r_2)$$
$$= 4(x - 1/4)(x - (-3))$$
$$= (4x - 1)(x + 3).$$