# Math 4330, Extra Credit, due 2/17 [1]

Define a function $f : \mathbb{N} \longrightarrow \mathbb{N}$ by

$$f(n) = \begin{cases} 3n+1, & \text{if } n \text{ is odd,} \\ n/2, & \text{if } n \text{ is even.} \end{cases}$$

Use the notation $f^k$ to denote the composition of $f$ with itself $k$ times, so that $f^2(n) = f(f(n))$, $f^3(n) = f(f(f(n)))$, and so on. There is a famous conjecture that for each positive integer $n$, the sequence $n, f(n), f^2(n), f^3(n), \ldots$ eventually reaches 1. For example, if $n = 6$ this sequence is $6, 3, 10, 5, 16, 8, 4, 2, 1, \ldots$.

For each positive integer $n$, let $\lambda_n$ be the least positive integer such that $f^{\lambda_n}(n) = 1$. From the example above, $\lambda_6 = 8$.

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**Extra credit (10 points)**

- Write Python code to determine the average of $\lambda_1, \lambda_2, \ldots, \lambda_{10000}$.
- Tweak your code to determine the average of $\lambda_1, \lambda_2, \ldots, \lambda_{100000}$. (You may need to determine several more such averages, so consider writing a function which takes a parameter $t$ and returns the average of $\lambda_1, \lambda_2, \ldots, \lambda_t$).
- Make a conjecture about the average value of $\lambda_1, \lambda_2, \ldots, \lambda_x$. (You may need to compute several more averages to make a reasonable conjecture, so feel free to do so).